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A new picture on the (3+1)D topological mass mechanism

O S Ventura[1,2,3], **R L P G Amaral**[4], **J V Costa**[4,5], **L O Buffon**[2,6] **and V E R Lemes**[7]

[1] Coordenadoria de Matemática, Centro Federal de Educação Tecnológica do Espírito Santo, Avenida Vitória 1729, Jucutuquara, Vitória-ES, 29040-333, Brazil
[2] Centro Universitário de Vila Velha, Rua comissário José Dantas de Mello 15, Boa Vista, Vila Velha-ES, 29102-770, Brazil
[3] Uniest, Av Brasil 110, Jardim América, Cariacica-ES, 29140-490, Brazil
[4] Instituto de Física, Universidade Federal Fluminense, 24210-340, Niterói-RJ, Brazil
[5] FAESA, Campus I, Rua Anselmo Serrat,199, Ilha Monte Belo, Vitória-ES, 29040-410, Brazil
[6] Escola Superior de Ciências da Santa Casa de Misericórdia de Vitória, Av Nossa Senhora da Penha 2190, Santa Luiza, Vitória-ES, 29045-402, Brazil
[7] Instituto de Física, Universidade do Estado do Rio de Janeiro, Rua São Francisco Xavier 524, Maracanã, Rio de Janeiro-RJ, 20550-013, Brazil

E-mail: ozemar@uvv.br, rubens@if.uff.br, costa@if.uff.br, lobuffon@terra.com.br and vitor@dft.if.uerj.br

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Abstract
We present a class of mappings between the fields of the Cremmer–Sherk and pure BF models in 4D. These mappings are established by two distinct procedures. First, a mapping of their actions is produced iteratively resulting in an expansion of the fields of one model in terms of progressively higher derivatives of the other model fields. Second, an exact mapping is introduced by mapping their quantum correlation functions. The equivalence of both procedures is shown by resorting to the invariance under field scale transformations of the topological action. Related equivalences in 5D and 3D are discussed. The mapping in (2+1)D from the Maxwell–Chern–Simons to pure Chern–Simons models is investigated from a similar perspective.

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1. Introduction

The search for ultraviolet renormalizable models has always been one of the most attractive and relevant aspects of quantum field theory. As is well known, the program of describing the electro-weak interactions in the language of QFT is based on the construction of the Higgs

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mechanism for mass generation of the vector bosons. However, this mechanism relies on the existence of a scalar particle, the Higgs boson, whose experimental evidence is still lacking.

In this context, the topological mechanism for mass generation is attractive, since it provides masses for the gauge vector bosons without the explicit introduction of new scalar fields. For example, in three-dimensional spacetime, the topological non-Abelian Chern–Simons term generates mass for the Yang–Mills fields while preserving the exact gauge invariance [1]. In four dimensions, the topological mass generation mechanism occurs in the case of an anti-symmetric tensorial field $B_{\mu\nu}$. It has been shown that the Cremmer–Sherk action gives a massive pole to the vector gauge field in the Abelian context. This model is described by the action [2]

$$S = \int \mathrm{d}^4x \left(-\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} + \frac{m}{4} \varepsilon^{\mu\nu\rho\sigma} F_{\mu\nu} B_{\rho\sigma} \right). \qquad (1.1)$$

Indeed, as was shown [3], this model exists only in the Abelian version. In fact, possible non-Abelian generalizations of the action (1.1) will necessarily require non-renormalizable couplings, as in [4], or the introduction of extra fields [5]. Anti-symmetric fields in four dimensions also deserve attention since they appear naturally by integrating out the fermionic degrees of freedom in favour of bosonic fields in bosonization approaches. The fermionic current turns out to be expressed in terms of derivatives of the tensorial field as a topologically conserved current. The coupling of this current to the gauge field leads to terms in the effective action similar to the last one in (1.1).

An important property of the three-dimensional Yang–Mills type actions, in the presence of the Chern–Simons term, was pointed out in [6], i.e., it can be cast in the form of a pure Chern–Simons action through a nonlinear covariant redefinition of the gauge connection [7]. The quantum consequences of this fact were investigated in the BRST framework yielding an algebraic proof of the finiteness of the Yang–Mills action with topological mass [8]. A generalization of this mapping to the Cremmer–Sherk's action was presented in [9] both in the Abelian and in the non-Abelian cases.

In this work, we generalize the recursive mapping between Cremmer–Sherk's action and the pure topological BF model presented in [9] by presenting the general mapping in terms of arbitrary parameters. With this the fields of one action are expressed as a series of progressively higher derivatives of the other model fields with a new parameter introduced at each step. This mapping is also established along a different line in which the propagators of one action are reproduced using a closed expression in terms of the other action fields. This closed expression depends on arbitrary functions. With this it may be expanded and the generic recursive mapping reobtained. This exposes the non-local nature of the mapping. Related mappings in higher and lower dimensions are discussed. The mapping from Maxwell–Chern–Simons to Chern–Simons models is analysed from a similar perspective.

2. Mapping the fields

The aim of this section is to establish the classical equivalence between the Cremmer–Sherk's action and the pure BF theory, i.e., the first action can be mapped to the second one through a redefinition of the gauge field. Following the same steps of the three-dimensional case [6], we search for a redefinition of the fields A_μ and $B_{\mu\nu}$ as a series in powers of $1/m$ in terms of the fields $\hat{A}_\mu$ and $\hat{B}_{\mu\nu}$ in such a way that the relation below is valid[8]:

$$\mathcal{S}_M(A) + \mathcal{S}_H(B) + \mathcal{S}_{\mathrm{BF}}(A, B) = \mathcal{S}_{\mathrm{BF}}(\widehat{A}, \widehat{B}), \qquad (2.1)$$

[8] We work in the Minkowski spacetime so that $\varepsilon^{\mu\nu\rho\sigma}\varepsilon_{\mu\nu\alpha\beta} = -2(\delta^\rho_\alpha\delta^\sigma_\beta - \delta^\rho_\beta\delta^\sigma_\alpha)$. We use $\varepsilon^{0123} = 1$ and diag $\eta_{\mu\nu} = (1, -1, -1, -1)$.

where

$$\mathcal{S}_M(A) = -\frac{1}{4}\int \mathrm{d}^4x(F^{\mu\nu}F_{\mu\nu}), \tag{2.2}$$

$$\mathcal{S}_H(B) = +\frac{1}{12}\int \mathrm{d}^4x(H_{\mu\nu\rho}H^{\mu\nu\rho}), \tag{2.3}$$

$$\mathcal{S}_{\mathrm{BF}}(A,B) = \frac{m}{4}\int \mathrm{d}^4x(\varepsilon^{\mu\nu\rho\sigma}F_{\mu\nu}B_{\rho\sigma}) \tag{2.4}$$

and the curvatures $F_{\mu\nu}$ and $H_{\mu\nu\rho}$ are the same given in (1.1), i.e.,

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

and

$$H_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} + \partial_\rho B_{\mu\nu} + \partial_\nu B_{\rho\mu}.$$

Indeed taking the field redefinitions in the form

$$\widehat{A}_\mu = A_\mu + \sum_{n=1}^{\infty}\frac{1}{(2m)^n}\vartheta_\mu^n, \qquad \widehat{B}_{\mu\nu} = B_{\mu\nu} + \sum_{n=1}^{\infty}\frac{1}{(2m)^n}\phi_{\mu\nu}^n, \tag{2.5}$$

the equality expressed in (2.1) is implemented by recursively fixing the terms. We find the expressions

$$\begin{aligned} \phi_{\mu\nu}^{2n+1} &= -\frac{b_{2n+1}}{2}\epsilon_{\mu\nu\alpha\beta}\Box^n F^{\alpha\beta}, \qquad & \vartheta_\mu^{2n+1} &= \frac{c_{2n+1}}{3}\epsilon_{\mu\nu\alpha\beta}\Box^n H^{\nu\alpha\beta}, \\ \phi_{\mu\nu}^{2n} &= b_{2n}\Box^{n-1}\partial^\alpha H_{\alpha\mu\nu}, & \vartheta_\mu^{2n} &= c_{2n}\Box^{n-1}\partial^\nu F_{\nu\mu}, \end{aligned} \tag{2.6}$$

where the constants are defined as

$$b_{2n+1} = -\sum_{j=1}^{n} c_{2j}b_{2(n-j)+1}, \tag{2.7}$$

$$c_{2n+1} = -\sum_{j=1}^{n} b_{2j}c_{2(n-j)+1}, \tag{2.8}$$

$$b_{2n} = B_n\left[2\sum_{j=1}^{n} b_{2j-1}c_{2(n-j)+1} - \sum_{j=1}^{n-1} b_{2j}c_{2(n-j)}\right], \tag{2.9}$$

$$c_{2n} = (1-B_n)\left[2\sum_{j=1}^{n} b_{2j-1}c_{2(n-j)+1} + \sum_{j=1}^{n-1} b_{2j}c_{2(n-j)}\right]. \tag{2.10}$$

Here B_n are arbitrary constants introduced at each even step of the process while $b_1 = -1$ and $c_1 = -1/2$.

The first terms can be expressed as

$$\begin{aligned} \vartheta_\mu^1 &= \frac{-1}{6}\varepsilon_{\mu\nu\alpha\beta}H^{\nu\alpha\beta}, \\ \vartheta_\mu^2 &= (1-B_1)\partial^\nu F_{\nu\mu}, \\ \vartheta_\mu^3 &= \frac{B_1}{6}\varepsilon_{\mu\nu\alpha\beta}\Box H^{\nu\alpha\beta}, \end{aligned}$$

$$
\begin{aligned}
\vartheta^4_\mu &= \left(-1 - B_1 + B_1^2\right)(1 - B_2)\,\Box\,\partial^\nu F_{\nu\mu},\\
\phi^1_{\mu\nu} &= -\frac{1}{2}\varepsilon_{\mu\nu\alpha\beta}F^{\alpha\beta},\\
\phi^2_{\mu\nu} &= \frac{B_1}{3}\partial^\alpha H_{\alpha\mu\nu},\\
\phi^3_{\mu\nu} &= -\frac{1-B_1}{2}\varepsilon_{\mu\nu\alpha\beta}\Box\,F^{\alpha\beta},\\
\phi^4_{\mu\nu} &= \frac{B_2\left(-1 - B_1 + B_1^2\right)}{3}\,\Box\,\partial^\alpha H_{\alpha\mu\nu}.
\end{aligned}
\tag{2.11}
$$

As we can see, up to the fourth order in the mass parameter, the coefficients, $\phi^n_{\mu\nu}$ and ϑ^n_μ, shown in (2.11), depend on the two arbitrary dimensionless parameters, B_1 and B_2. In fact at each new even order in $1/m$, a new arbitrary parameter is allowed to be introduced. As we shall see, this is to be expected.

The formal series (2.5), which redefine the fields A_μ and $B_{\mu\nu}$, give the mapping we were looking for.

Note that the gauge symmetry of the Cremmer–Sherk action is expressed as

$$\delta^g A_\mu = \partial_\mu\varepsilon, \qquad \delta^g B_{\mu\nu} = 0 \tag{2.12}$$

and

$$\delta^t A_\mu = 0, \qquad \delta^t B_{\mu\nu} = \partial_\mu\varepsilon_\nu - \partial_\nu\varepsilon_\mu, \tag{2.13}$$

while the BF topological action is invariant under analogous transformations

$$\delta^g \hat{A}_\mu = \partial_\mu\hat{\varepsilon}, \qquad \delta^g \hat{B}_{\mu\nu} = 0 \tag{2.14}$$

and

$$\delta^t \hat{A}_\mu = 0, \qquad \delta^t \hat{B}_{\mu\nu} = \partial_\mu\varepsilon_\nu - \partial_\nu\hat{\varepsilon}_\mu. \tag{2.15}$$

The mapping (2.5) translates the gauge transformations of one pair of fields straightforwardly into the ones of the other pair in such a way that $\hat{\varepsilon}$ and $\hat{\varepsilon}_\mu$ are identified as ε and ε_μ. This occurs since the higher order terms in (2.6) have been chosen to be gauge invariant.

2.1. Exact mapping

Let us express here the Cremmer–Sherk fields in terms of the pure BF fields using a new procedure. It will be convenient to change the parameter m of the pure BF action to $\hat{m}$. The Cremmer–Sherk propagators are given by

$$
\begin{aligned}
\langle \imath\mathcal{T} B_{\mu\nu} B_{\alpha\beta}\rangle &= (P_{\mu\nu,\alpha\beta} + G_1 K_{\mu\nu,\alpha\beta})\frac{2}{\Box(\Box + m^2)},\\
\langle \imath\mathcal{T} B_{\mu\nu} A_\alpha\rangle &= -\langle \imath\mathcal{T} A_\alpha B_{\mu\nu}\rangle = (S_{\mu\nu\alpha})\frac{m}{\Box(\Box + m^2)},\\
\langle \imath\mathcal{T} A_\mu A_\alpha\rangle &= (P_{\mu,\nu} + G_2 K_{\mu,\nu})\frac{-1}{\Box(\Box + m^2)}.
\end{aligned}
\tag{2.16}
$$

The pure BF propagators are given by

$$
\begin{aligned}
\langle \imath\mathcal{T} \widehat{B}_{\mu\nu} \widehat{B}_{\alpha\beta}\rangle &= (\widehat{G}_1 K_{\mu\nu,\alpha\beta})\frac{2}{\hat{m}\,\Box},\\
\langle \imath\mathcal{T} \widehat{B}_{\mu\nu} \widehat{A}_\alpha\rangle &= -\langle \imath\mathcal{T} \widehat{A}_\alpha \widehat{B}_{\mu\nu}\rangle = (S_{\mu\nu\alpha})\frac{1}{\hat{m}\,\Box},\\
\langle \imath\mathcal{T} \widehat{A}_\mu \widehat{A}_\nu\rangle &= (\widehat{G}_2 K_{\mu,\nu})\frac{1}{\hat{m}\,\Box}.
\end{aligned}
\tag{2.17}
$$

Here the projectors are given by

$$
\begin{aligned}
P_{\mu\nu,\alpha\beta} &= \tfrac{1}{2}\delta_{\mu\nu,\alpha\beta}\Box - \tfrac{1}{2}K_{\mu\nu,\alpha\beta},\\
K_{\mu\nu,\alpha\beta} &= \delta_{\mu[\alpha}\partial_\nu\partial_{\beta]} - \delta_{\nu[\alpha}\partial_\mu\partial_{\beta]},\\
S_{\mu\nu\alpha} &= \varepsilon_{\mu\nu\alpha\beta}\partial^\beta,\\
P_{\mu\nu} &= \delta_{\mu\nu}\Box - \partial_\mu\partial_\nu,\\
K_{\mu\nu} &= \partial_\mu\partial_\nu,
\end{aligned}
\tag{2.18}
$$

where $\delta_{\mu\nu,\alpha\beta} = \delta_{\mu\alpha}\delta_{\nu\beta} - \delta_{\mu\beta}\delta_{\nu\alpha}$. The parameters G and $\widehat{G}$ are introduced to fix the gauge.

Let us try to express the fields as

$$
\begin{aligned}
A_\mu &= \left(C_{AA}P_{\mu,\nu} + \sqrt{\frac{\hat{m}}{m}}\delta_{\mu\nu}\right)\widehat{A}^\nu + C_{AB}S_{\mu\alpha\beta}\widehat{B}^{\alpha\beta},\\
B_{\mu\nu} &= C_{BA}S_{\mu\nu\alpha}\widehat{A}^\nu + \left(C_{BB}P_{\mu\nu,\alpha\beta} + \frac{1}{2}\sqrt{\frac{\hat{m}}{m}}\delta_{\mu\nu,\alpha\beta}\right)\widehat{B}^{\alpha\beta}.
\end{aligned}
\tag{2.19}
$$

Computing the correlators of the Cremmer–Sherk field using this mapping and comparing with (2.16), the structure functions are fixed. They turn out to be given by the non-local operators

$$
\begin{aligned}
C_{AA} &= \frac{2^{\frac{1}{2}}\hat{m}^{\frac{1}{2}}}{\sigma\Box}\left[\frac{m-\sqrt{m^2+\Box}}{m^2+\Box}\right]^{\frac{1}{2}} - \sqrt{\frac{\hat{m}}{m}}\frac{1}{\Box},\\
C_{BB} &= \frac{\sigma\hat{m}^{\frac{1}{2}}}{2^{\frac{3}{2}}\Box}\left[\frac{m-\sqrt{m^2+\Box}}{m^2+\Box}\right]^{\frac{1}{2}} - \sqrt{\frac{\hat{m}}{m}}\frac{1}{\Box},\\
C_{AB} &= \frac{\sigma\hat{m}^{\frac{1}{2}}}{2^{\frac{5}{2}}}\left[\left(m-\sqrt{m^2+\Box}\right)(m^2+\Box)\right]^{-\frac{1}{2}},\\
C_{BA} &= \frac{2^{\frac{1}{2}}\hat{m}^{\frac{1}{2}}}{\sigma}\left[\left(m-\sqrt{m^2+\Box}\right)(m^2+\Box)\right]^{-\frac{1}{2}}.
\end{aligned}
\tag{2.20}
$$

Note that the non-local operators indeed map local fields of local models, the Cremmer–Sherk and pure BF models. Observe the presence of the arbitrary operator σ in these equations. Its presence should be expected since the set of transformations

$$
\hat{A} \longrightarrow \sigma\hat{A}, \qquad \hat{B} \longrightarrow \frac{1}{\sigma}\hat{B},
\tag{2.21}
$$

does not affect any correlator of the BF model. The presence of σ in the mapping is due to the freedom in redefining the BF fields. This is the ultimate reason for the presence of the free parameters $(B_1, B_2\ldots)$ in the mapping seen previously. In fact the exact inverse mapping is given by

$$
\begin{aligned}
\widehat{A}_\mu &= \left(\widehat{C}_{AA}P_{\mu,\nu} + \sqrt{\frac{m}{\hat{m}}}\delta_{\mu\nu}\right)A^\nu + \widehat{C}_{AB}S_{\mu\alpha\beta}B^{\alpha\beta},\\
\widehat{B}_{\mu\nu} &= \widehat{C}_{BA}S_{\mu\nu\alpha}A^\nu + \left(\widehat{C}_{BB}P_{\mu\nu,\alpha\beta} + \sqrt{\frac{m}{\hat{m}}}\frac{1}{2}\delta_{\mu\nu,\alpha\beta}\right)B^{\alpha\beta},
\end{aligned}
\tag{2.22}
$$

with

$$\widehat{C}_{AA} = \frac{\sigma}{2^{\frac{3}{2}}\hat{m}^{\frac{1}{2}}\Box}\left[\frac{(m-\sqrt{m^2+\Box})^{\frac{3}{2}}}{(m^2+\Box)^{\frac{1}{2}}}\right] - \sqrt{\frac{m}{\hat{m}}}\frac{1}{\Box},$$
$$\widehat{C}_{BB} = \frac{2^{\frac{1}{2}}}{\sigma\hat{m}^{\frac{1}{2}}\Box}\left[\frac{(m-\sqrt{m^2+\Box})^{\frac{3}{2}}}{(m^2+\Box)^{\frac{1}{2}}}\right] - \sqrt{\frac{m}{\hat{m}}}\frac{1}{\Box},$$
$$\widehat{C}_{AB} = \frac{\sigma}{2^{\frac{5}{2}}\hat{m}^{\frac{1}{2}}}\left[\frac{(m-\sqrt{m^2+\Box})}{(m^2+\Box)}\right]^{\frac{1}{2}},$$
$$\widehat{C}_{BA} = \frac{2^{\frac{1}{2}}}{\sigma\hat{m}^{\frac{1}{2}}}\left[\frac{(m-\sqrt{m^2+\Box})}{(m^2+\Box)}\right]^{\frac{1}{2}}. \tag{2.23}$$

The iterative mapping may be retrieved by expanding the structure functions in terms of $\frac{\Box}{m^2}$ and, at the same time, expressing the operator σ in terms of arbitrary parameters as

$$\sigma = \sum_{n=0}^{\infty} C_n \left(\frac{\Box}{m^2}\right)^n \tag{2.24}$$

and similarly to its inverse. With this equation (2.22) will reproduce equations (2.11) for $\hat{m} = m$. The independent parameters B_j are thus seen to owe their origin to the freedom in defining the operator σ. This allows for an independent parameter, C_j, to be introduced at each order in $\Box^j$.

The structure functions can be alternatively expanded in powers of m instead of powers of $1/m$. Particularly important is to consider the limit $m \longrightarrow 0$ in equations (2.23). This leads to the mapping from the fields of a model without topological terms to the purely topological model fields:

$$\widehat{C}_{AA} = \frac{\sigma}{2^{\frac{3}{2}}\hat{m}^{\frac{1}{2}}}\Box^{-\frac{3}{4}}, \qquad \widehat{C}_{BB} = -\frac{2^{\frac{1}{2}}}{\sigma\hat{m}^{\frac{1}{2}}}\Box^{-\frac{3}{4}},$$
$$\widehat{C}_{AB} = \frac{-\sigma}{2^{\frac{5}{2}}\hat{m}^{\frac{1}{2}}}\Box^{-\frac{1}{4}}, \qquad \widehat{C}_{BA} = \frac{2^{\frac{1}{2}}}{\sigma\hat{m}^{\frac{1}{2}}}\Box^{-\frac{1}{4}}. \tag{2.25}$$

The inverse of this mapping is easily obtained. It can also be obtained by performing the limit $m \longrightarrow 0$ directly in (2.20) provided that the term with $1/\Box$ is first reabsorbed in equation (2.19) by eliminating the identity term in this equation. This amounts to a change in the gauge fixing conditions. The series expansions in powers of m of (2.23) turn out to be a set of series in powers of $m/\sqrt{\Box}$ multiplying its zeroth-order expressions (2.25). These series can be alternatively obtained in a procedure that parallels the one used to obtain the iterative mapping in powers of $\Box/m^2$. For this (2.25) should be taken as the zeroth-order expression that maps the action $\mathcal{S}_M(A) + \mathcal{S}_H(B)$ to $\mathcal{S}_{\mathrm{BF}}(\hat{A}, \hat{B})$. Now the perturbation $\mathcal{S}_{\mathrm{BF}}(A, B)$ is taken into account and its contribution is cancelled at each step of the process with higher order terms. Comparing to the previous procedures the roles of the kinetic terms are thus reversed. The presence of $\sqrt{\Box}$ in these series may seem suspicious at first sight. After all if the propagators of the Cremmer–Sherk model fields are expanded in powers of m they turn out to produce a series of $m^2/\Box$. Indeed, an explicit computation shows that when the series obtained by expanding (2.20) is used to obtain the propagators of A and B from the ones of $\hat{A}$ and $\hat{B}$ the terms with square roots of the D'Alembertian cancel out.

3. Dimensional reduction considerations

Let us consider in 5D the model with the anti-symmetric field which represents a direct generalization of the Maxwell–Chern–Simons model [10, 11]

$$S = \int \mathrm{d}^5x \left(\frac{1}{6} H^*_{\mu\nu\rho} H^{\mu\nu\rho} + \imath \frac{m}{24} \varepsilon^{\mu\nu\alpha\beta\rho} (B^*_{\mu\nu} H_{\alpha\beta\rho} - B_{\mu\nu} H^*_{\alpha\beta\rho}) \right). \tag{3.1}$$

The mapping of the field to the pure Chern–Simons field

$$\widehat{S} = \imath \int \mathrm{d}^5x \left(\frac{m}{24} \varepsilon^{\mu\nu\alpha\beta\rho} (B^*_{\mu\nu} H_{\alpha\beta\rho} - B_{\mu\nu} H^*_{\alpha\beta\rho}) \right) \tag{3.2}$$

is implemented by the transformation

$$\begin{aligned} B_{\mu\nu} = \hat{B}_{\mu\nu} &+ \left[\frac{m^{\frac{1}{2}}}{2^{\frac{1}{2}} \Box} \left[\frac{(m - \sqrt{m^2 + \Box})^{\frac{1}{2}}}{(m^2 + \Box)^{\frac{1}{2}}} \right] - \frac{1}{\Box} \right] P_{\mu\nu,\alpha\beta} \widehat{B}^{\alpha\beta} \\ &- \imath \frac{m^{\frac{1}{2}}}{2^{\frac{1}{2}} 6} \left[\frac{(m - \sqrt{m^2 + \Box})}{(m^2 + \Box)} \right]^{\frac{1}{2}} \varepsilon_{\mu\nu\alpha\beta\rho} \widehat{H^{\alpha\beta\rho}}. \end{aligned} \tag{3.3}$$

This result is obtained repeating the argument of section 2b in the five-dimensional spacetime. Note that in this case the topological model does not present any freedom in rescaling the fields as occurs in 4D. The dimensional reduction of model (3.1) by precluding any dependence on the variable x^4 so that $B_{\mu 4} := A_\mu$ leads to complex fields Cremmer–Sherk models [10]

$$\mathcal{L}_M(B) = -\frac{1}{2} F^{*\mu\nu} F_{\mu\nu} + \frac{1}{6} H^*{}_{\mu\nu\rho} H^{\mu\nu\rho} - \imath \frac{m}{4} \varepsilon^{\mu\nu\rho\sigma} (F^*{}_{\mu\nu} B_{\rho\sigma} - F_{\mu\nu} B^*{}_{\rho\sigma}). \tag{3.4}$$

Expressed in terms of real and imaginary components, this Lagrangian is related to a couple of models in (1.1). Under similar considerations, the topological model (3.2) is led to the pure (complex) BF model (2.4) or to a pair of real BF models. Within this setting, the mapping of the 5D fields (3.3) is reduced to the exact mapping in equations (2.19) and (2.20) if we eliminate the freedom in the 4D mapping by identifying $\sigma = 2$. Thus, the dimensional reduction turns out to give a criterion to fix in a natural fashion the mapping of the fields.

As we have seen, the mapping connecting the model with topological mass generation in four dimensions to the pure topological BF model is related to similar properties of models in five-dimensional spacetime which present only the anti-symmetric field. In this section we perform one more step in the dimensional reduction programme presenting the similar property appearing in the model obtained after the dimensional reduction of the (real field) Cremmer–Sherk's action to 3D. The reduced action is given by

$$\begin{aligned} \mathcal{S} &= \mathcal{S}_{\mathrm{top}} + \mathcal{S}_{n\mathrm{top}} \\ &= \int \mathrm{d}^3x \left(-\frac{m}{6} \varepsilon^{\mu\nu\rho} \varphi H_{\mu\nu\rho} + \frac{m}{2} \varepsilon^{\mu\nu\rho} c_\mu F_{\nu\rho} \right) \\ &\quad + \int \mathrm{d}^3x \left(-\frac{1}{4} G^{\mu\nu} G_{\mu\nu} - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} + \frac{1}{2} \partial^\mu \varphi \partial_\mu \varphi \right), \end{aligned}$$

where, after the reduction,

$$A_\mu \to A_{\mu,}\varphi, \qquad B_{\mu\nu} \to B_{\mu\nu}, C_\mu, \qquad G_{\mu\nu} = \partial_\mu C_\nu - \partial_\nu C_\mu.$$

The mapping to ensure that

$$\mathcal{S}_{\mathrm{top}}(\widehat{A}_\mu, \widehat{B}_{\mu\nu}, \widehat{\varphi}, \widehat{c}_\mu) = \mathcal{S}_{\mathrm{top}}(A_\mu, B_{\mu\nu}, \varphi, c_\mu) + \mathcal{S}_{n\mathrm{top}}(A_\mu, B_{\mu\nu}, \varphi, c_\mu),$$

will be given by

$$\widehat{A}_\mu = A_\mu + \sum_{n=1}^{\infty} \frac{1}{(2m)^n} \vartheta_\mu^n, \tag{3.5}$$

$$\widehat{B}_{\mu\nu} = B_{\mu\nu} + \sum_{n=1}^{\infty} \frac{1}{(2m)^n} \phi_{\mu\nu}^n, \tag{3.6}$$

$$\widehat{\varphi} = \varphi + \sum_{n=1}^{\infty} \frac{1}{(2m)^n} \alpha^n, \tag{3.7}$$

$$\widehat{c}_\mu = c_\mu + \sum_{n=1}^{\infty} \frac{1}{(2m)^n} k_\mu^n. \tag{3.8}$$

Following the same lines as in the previous section the coefficients are easily obtained.

It should be clear that the above expression may be summed up leading to expressions that parallel the exact mapping obtained in $D = 4$. Indeed, the classical argument of mapping the actions as depicted above allows us to obtain the coefficients in dimensionally reduced models from the knowledge of the higher dimensional mapping coefficients. Since the propagators are the inverses of the operators defining the actions, their exact mapping in lower dimensions can also be read from the corresponding expressions in higher dimensions. The non-local operators that map the local fields are essentially the same ones.

4. Maxwell–Chern–Simons model

Let us consider now in 3D the generic model described by the Lagrangian

$$\mathcal{L}^1 = -\frac{\eta}{2m} A_\mu P^{\mu\nu} \mathcal{C} A_\nu + \frac{\eta}{2} A_\mu S^{\mu\nu} \mathcal{D} A_\nu + \frac{1}{2} A_\mu K^{\mu\nu} \alpha A_\nu. \tag{4.1}$$

Here $\mathcal{C}$ and $\mathcal{D}$ are arbitrary scalar operators, $P^{\mu\nu} = \Box\delta^{\mu\nu} - \partial^\mu\partial^\nu$, $S^{\mu\nu} = \epsilon^{\mu\nu\alpha}\partial_\alpha$ and the term with $K^{\mu\nu} = \partial^\mu\partial^\nu$ corresponds to a gauge fixing term.

The two-point function for this field is obtained as

$$G_{\mu\nu} = \mathrm{i}\langle \mathcal{T} A_\mu A_\nu \rangle = P_{\mu\nu} \frac{m\mathcal{C}}{\eta(m^2\mathcal{D}^2\Box + \mathcal{C}^2\Box^2)} - S_{\mu\nu} \frac{m^2\mathcal{D}}{\eta(m^2\mathcal{D}^2\Box + \mathcal{C}^2\Box^2)} + K_{\mu\nu}\gamma. \tag{4.2}$$

We perform the mapping

$$A^\mu = \hat{f} S^{\mu\nu} \widehat{A}_\nu + \hat{g} P^{\mu\nu} \widehat{A}_\nu + \hat{k} K^{\mu\nu} \widehat{A}_\nu, \tag{4.3}$$

where the action for the field $\widehat{A}_\mu$ is the Chern–Simons action so that

$$\langle \imath \mathcal{T} \widehat{A}_\mu \widehat{A}_\nu \rangle = -S_{\mu\nu} \frac{1}{\Box} + K_{\mu\nu}\hat{\gamma}. \tag{4.4}$$

Using the algebra $P^2 = \Box P$, $PS = \Box S$ and $S^2 = -P$ it is readily obtained that

$$\langle \imath \mathcal{T} A_\mu A_\nu \rangle = (-\hat{g}^2\Box + \hat{f}^2) S_{\mu\nu} + 2\hat{f}\hat{g} P_{\mu\nu} + \hat{k}\hat{\gamma}\Box^2 K_{\mu\nu}. \tag{4.5}$$

The propagators in (4.2) and (4.5) are indeed identical if the form factors satisfy

$$\hat{g} = \sqrt{\frac{m}{2\eta}} \mathcal{C} \left[\Box(m^2\mathcal{D}^2 + \mathcal{C}^2\Box)\big(\epsilon(\Box\mathcal{C}^2 + m^2\mathcal{D}^2)^{\frac{1}{2}} - m\mathcal{D}\big) \right]^{-\frac{1}{2}}, \tag{4.6}$$

and

$$\hat{f} = \sqrt{\frac{m}{2\eta}} \left[\frac{\epsilon(\Box \mathcal{C}^2 + m^2\mathcal{D}^2)^{\frac{1}{2}} - m\mathcal{D}}{\Box(m^2\mathcal{D}^2 + \Box \mathcal{C}^2)} \right]^{\frac{1}{2}} \tag{4.7}$$

where $\epsilon = \pm 1$ controls the different branches of the square root. We thus obtain the exact mapping from a field with the action described by (4.1) to a field with the pure Chern–Simmons action by means of a mapping with rather involved non-local structure functions.

Let us particularize to some interesting limiting cases. First consider the case $\mathcal{C} = \mathcal{D} = 1$. The original action corresponds to the Maxwell–Chern–Simons model. The mapping to the pure Chern–Simons field is implemented by the structure functions:

$$\hat{g} = \sqrt{\frac{m}{2\eta}} \left[\frac{m + \epsilon(\Box + m^2)^{\frac{1}{2}}}{\Box^2(m^2 + \Box)} \right]^{1/2} \tag{4.8}$$

$$\hat{f} = \sqrt{\frac{m}{2\eta}} \left[\frac{-m + \epsilon(\Box + m^2)^{\frac{1}{2}}}{\Box(m^2 + \Box)} \right]^{1/2}. \tag{4.9}$$

Taking the limit where $\eta = m \longrightarrow 0$ with $\epsilon = 1$ leads the original action to the pure Maxwell field action. This will be used in the following to establish a series in powers of m. It is readily obtained that the limiting structure functions become the much simpler non-local operators [12]

$$\hat{g} = \frac{1}{\sqrt{2}} \Box^{-\frac{5}{4}} \tag{4.10}$$

$$\hat{f} = \frac{1}{\sqrt{2}} \Box^{-\frac{3}{4}}. \tag{4.11}$$

Next consider the non-local vectorial model which results from the bosonization of the free fermionic field [13]. In this case we consider $\mathcal{C} = \Box^{-1/2}$ and $\mathcal{D} = 1$ so that

$$\mathcal{L}^1 = \frac{\eta}{4m} F_{\mu\nu} \Box^{-1/2} F^{\mu\nu} + \frac{\eta}{2} A_\mu S^{\mu\nu} A_\nu + \frac{1}{2} A_\mu K^{\mu\nu} \alpha A_\nu. \tag{4.12}$$

Note that in this case the parameter m is dimensionless. The structure functions then become

$$\hat{g} = \sqrt{\frac{m}{2\eta}} \Box^{-1} \left[\left(-m + \epsilon(1 + m^2)^{\frac{1}{2}}\right)(m^2 + 1) \right]^{-1/2} \tag{4.13}$$

$$\hat{f} = \sqrt{\frac{m}{2\eta}} \Box^{-1/2} \left[\frac{-m + \epsilon(1 + m^2)^{\frac{1}{2}}}{m^2 + 1} \right]^{1/2}. \tag{4.14}$$

Some limiting cases of this last non-local model are worth considering explicitly. If $m \longrightarrow \infty$, the original action is already the Chern–Simons action. The structure function $\hat{f} = 0$ while $\hat{g} = \Box/\sqrt{2}$ are obtained with the signal of the square root signalized by $\epsilon = 1$. Using the freedom in defining the gauge fixing term, this implements the identity mapping of the fields A and $\widehat{A}$. The same limit taken with the alternative signal of the square root with $\epsilon = -1$ leads instead to $\hat{g} = 0$ and

$$\hat{f} = \frac{-\imath}{\eta\sqrt{\Box}}, \tag{4.15}$$

and this implements a mapping from a vectorial to pseudo-vectorial field both with Chern–Simons action.

In the following we consider the Maxwell–Chern–Simons model, $\mathcal{C} = \mathcal{D} = 1$ and obtain the iterative solutions. First we consider the infrared expansion. We expand the structure functions in powers of $\Box/m^2$ and obtain

$$\hat{g} = \frac{1}{\sqrt{\eta}\,\Box}\left[1 - \frac{3\,\Box}{2m^2} + \frac{35\,\Box^2}{128m^4} - \frac{231\,\Box^3}{1024m^6} + \frac{6435\,\Box^4}{32960m^8} + \cdots\right] \tag{4.16}$$

$$\hat{f} = \frac{1}{2m\sqrt{\eta}}\left[1 - \frac{5\,\Box}{2^3m^2} + \frac{63\,\Box^2}{2^7m^4} - \frac{429\,\Box^3}{2^{10}m^6} + \cdots\right]. \tag{4.17}$$

This series has been obtained in [6] through an iterative procedure similar to the one we used in the four-dimensional case. The exact mapping displays the non-local feature of the structure functions which may be somewhat masked in the direct iterative procedure.

Next we consider the series in the opposite ultraviolet limit, expanding in powers of $y = m/\sqrt{\Box}$. We see that

$$\hat{g} = \sqrt{\frac{m}{2\eta}}\,\Box^{-\frac{5}{4}}\left[1 + \frac{1}{2}\frac{m}{\sqrt{\Box}} - \frac{3}{8}\left(\frac{m}{\sqrt{\Box}}\right)^2 - \frac{5}{16}\left(\frac{m}{\sqrt{\Box}}\right)^3 + \cdots\right] \tag{4.18}$$

and that

$$\hat{f} = \sqrt{\frac{m}{2\eta}}\,\Box^{-\frac{3}{4}}\left[1 - \frac{1}{2}\frac{m}{\sqrt{\Box}} - \frac{3}{8}\left(\frac{m}{\sqrt{\Box}}\right)^2 + \frac{5}{16}\left(\frac{m}{\sqrt{\Box}}\right)^3 + \cdots\right]. \tag{4.19}$$

Note the interesting property that although the mapping of the field involves odd and even powers of y when computing the correlation functions for the field using the series expansion only even powers of y persist. Indeed this comes out since the series for $\hat{f}$ and $\hat{g}$ differ, essentially, for the signals of the odd-order terms. Then computing the propagator explicitly, using equations (4.18) and (4.19) in equation (4.5) and disregarding the gauge-dependent term, leads to

$$G_{\mu\nu} = \mathrm{i}\langle \mathcal{T} A_\mu A_\nu\rangle = \frac{m}{\eta\,\Box^2}(P_{\mu\nu} - mS_{\mu\nu})\left(1 - \frac{m^2}{\Box} + \frac{m^4}{\Box^2} + \cdots\right). \tag{4.20}$$

This reproduces the series in powers of $m^2/\Box$ of the correlator of the MCS model (4.2).

Let us also comment that the (ultraviolet) series in powers of y, quite similar to the 4D case, can also be obtained recursively in a procedure entirely analogous to the one used in [6] to obtain the infrared series. First we should consider the Maxwell term in the MCS model. The CS action for the $\widehat{A}$ field is retrieved from this term alone by considering the first terms in ultraviolet expansion for both $\hat{f}$ and $\hat{g}$. This amounts to considering the mapping from pure Maxwell action to the pure Chern–Simons action we obtained before. Next, introducing perturbations in both expressions and incorporating the Chern–Simons term of the MCS model, one is led to the series in powers of y obtaining the terms recursively by imposing to the $\widehat{A}$ field the preservation of the CS action. The series in powers of $m/\sqrt{\Box}$ emerges from the lower derivative power in the Chern–Simons term compared to the Maxwell term.

5. Conclusions and discussions

In this work, we have studied the procedure that allows us to map the Maxwell–Chern–Simons field to the pure Chern–Simons field in (2+1)D, and in (3+1)D the Cremmer–Sherk model, to the Abelian version of the BF model. A striking importance of these mappings stems from the fact that the dynamical mass mechanism, which occurs in the topologically massive models, can be described in the context of purely topological models. The latter models presenting physical contents remarkably distinct from the former ones may offer new insights on this physical mechanism. Besides, the possibility of obtaining the Green functions of the topologically massive models from those of the topological models which present scale invariance may offer valuable computational advantages.

The four-dimensional mapping has been established firstly within an iterative general procedure. One remarkable new aspect that emerges is the presence of a great deal of freedom in the mapping in four dimensions. This freedom has been elucidated as due to the form of the pure topological action which is defined through mixed products of fields. The invariance under rescaling of the fields of the BF type action is responsible for it. Since this kind of action is naturally considered in even dimensional topological models, the non-uniqueness in the mapping should be expected to hold in even dimensions.

The exact mappings of the fields, both in four and in three dimensions, have been presented. The exact mapping may be established even in the cases in which the topological terms are absent. The topologically massive cases allows for series expansions of the exact mappings. The knowledge of the exact mappings provides us with a typical scale, given by the mass parameter m. The mapping may be used for instance for computing loop variables of the Cremmer–Sherk model using the corresponding expressions of the pure BF model. This suggests performing the computation in closed fashion without resource to expansions given by the iterative mapping. In any case the mass parameter m may provide valuable hints to discern in which cases computations using the iterative mapping should or should not be considered reliable. It can even provide alternative expansions for instance in direct powers of m instead of the inverse power series provided by the iterative mapping.

In the three-dimensional case indeed, the mapping from the Maxwell–Chern–Simons model to the Chern–Simons model, the knowledge of the explicit non-iterative expression for the structure functions allows us to revisit the computation of the link invariants for the MCS model from the corresponding expression for the pure CS field addressed in [6]. The explicit expression allows one to further understand the limits under which the iterative mapping should be considered. Indeed, the complete non-iterative expression is highly non-local while any truncation of the series involves essentially a local expression with the order of the derivative dependent on the order of the polynomial expression resulting from the series truncation. This explains why it should not be expected that expectation values of loop variables for non-intercepting curves with the MCS model field should have the same values as the ones computed with the pure CS field for arbitrary loops. The null character of the contribution of each iterative term to the corrections to the CS field loop correlations coming from the Maxwell term obtained correctly in [6] is dependent on the local character of the truncated mapping. It should not be confused with assertions to the vanishing of the exact result contribution. The non-locality of the exact mapping is responsible for non-null contributions. This result that each series term contribution is null while the 'summed series' gives non-null contributions signalizes that an expansion in powers of $x = \Box/m^2$ is not the proper one for doing the computation of general loop variables correlation functions. In this sense, it seems one is trying to perform an expansion in powers of x of a function in which all derivatives are zero for $x = 0$. It does not mean the function is identically zero but only that the function cannot be expanded in this series. It is thus much more reliable to deal with the opposite series in powers of y we presented here.

The dimensional reduction arguments presented here relates the mechanism of mapping from more involved actions to structurally minimized models in different dimensions. Besides providing a criterion to fix the mapping, the kinematical dimensional reduction may offer insights as to the low momenta field variables needed in dimensional reductions of high temperature limits in field theory. Actions of the Cremmer–Sherk type are expected to play a role in approaches where current fermions condensates are explicitly controlled with a bosonization scheme. The high temperature limit of QED under this setting will lead to a dimensional reduction paralleling the one provided here.

In order to properly appreciate the physical meaning of the mapping, it is important to call attention to the necessity of defining the physical content of a local field theory in terms of the local polynomial algebra of observable fields. The mapping provided here relates two local models each with its physical Hilbert space reconstructed from the Wightmann functions of its own polynomial algebra [14, 15]. Since the mapping involves non-local functions it should be clear that within the pure BF model there are two Hilbert spaces to be obtained. One Hilbert space is obtained from the local polynomial algebra of fields defined after expressing the Cremmer–Sherk fields non-locally in terms of the pure BF model fields and it should not be confused with the Hilbert space of the pure BF model itself. This later is obtained from its local polynomial algebra of fields. Although constructed with the same model fields, the first Hilbert space is not isomorphic to the second one. Instead, it will be isomorphic to the Hilbert space of the Cremmer–Sherk model. The same reasoning goes in the other direction of the mapping. In this context it is clear that neither Hilbert space should be considered as a subspace of the other. It is not a mapping of physical states that is being addressed here but a non-local mapping among the fields.

The generality of the mapping considered in the four-dimensional case can be further enhanced by introducing arbitrary scalar operators in the definitions of the quadratic non-mixed terms of the vector and anti-symmetric fields and considering the parameter m as a scalar operator acting either on vector or anti-symmetric field similar to what was done in the three-dimensional cases. This generalized gauge invariant action will be mapped to the pure BF model in a very similar way with the structure functions of the mapping being slightly modified. Furthermore, it is to be expected [8] that the introduction of arbitrary gauge invariant interaction terms can be absorbed by considering nonlinear mappings.

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